

# *Upfront Maths*

Correlation with The National Curriculum for England

Programme of Study: Mathematics

Key Stage 3, Key Stage 4 Foundation and Key Stage 4 Higher

# Key Stage 3

## Knowledge, skills and understanding

*Upfront* content draws connections between the sections on **number and algebra**, **shape, space and measures**, and **handling data**.

## Ma2 Number and algebra

### Using and applying number and algebra

*Upfront* provides pupils with the opportunity to:

#### Problem Solving

- a develop flexible approaches to increasingly demanding problems and select appropriate strategies to use for numerical or algebraic problems
- b breaking down a complex calculation into simpler steps before attempting to solve it (this is facilitated by adding procedural clues within the question)
- d select efficient techniques for numerical calculation and algebraic manipulation

#### Communicating

- f identify representations of problems and solutions in algebraic or graphical forms; move from one form of representation to another to get different perspectives on the problem; interpret solutions in the context of the original problem
- g develop correct and consistent use of notation, symbols and diagrams when solving problems

#### Reasoning

- i explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further
- j show step-by-step deduction in solving a problem; showing how they arrived at a conclusion

## Numbers and the number system

*Upfront* provides pupils with the opportunity to:

#### Integers

- a use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative numbers, both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition

#### Powers and roots

- b understand the terms square, positive and negative square root (knowing that the square root sign denotes the positive square root), cube, cube root; use index notation for small integer powers and index laws for multiplication and division of positive integer powers

#### Fractions

- c use fraction notation; understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator

## Decimals

- d use decimal notation and recognise that each terminating decimal is a fraction [for example,  $0.137 = 137/1000$ ]; order decimals

## Percentages

- e understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of' [for example, 10% means 10 parts per 100 and 15% of Y means  $15/100 \times Y$ ]

## Ratio and proportion

- f use ratio notation, including reduction to its simplest form and its various links to fraction notation
- g recognise where fractions or percentages are needed to compare proportions; identify problems that call for proportional reasoning, and choose the correct numbers to take as 100%, or as a whole

## Calculations

*Upfront* provides pupils with the opportunity to:

### Number operations and relationships between them

- a add, subtract, multiply and divide integers and then any number; multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1
- b understand brackets and the hierarchy of operations; know how to use the commutative, associative and distributive laws to do mental and written calculations more efficiently
- c calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal
- d understand and use unit fractions as multiplicative inverses [for example, by thinking of multiplication by  $1/5$  as division by 5, or multiplication by  $6/7$  as multiplication by 6 followed by division by 7 (or vice versa)]; multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction
- e convert simple fractions of a whole to percentages of the whole and vice versa, then understand the multiplicative nature of percentages as operators [for example, 20% discount on £150 gives a total calculated as  $£(0.8 \times 150)$ ]
- f divide a quantity in a given ratio [for example, share £15 in the ratio 1:2]

### Mental methods

- g recall all positive integer complements to 100 [for example,  $37 + 63 = 100$ ]; recall all multiplication facts to  $10 \times 10$ , and use them to derive quickly the corresponding division facts; recall the cubes of 2, 3, 4, 5 and 10, and the fraction-to-decimal conversion of familiar simple fractions [for example,  $1/2$ ,  $1/4$ ,  $1/5$ ,  $1/10$ ,  $1/100$ ,  $1/3$ ,  $2/3$ ,  $1/8$ ]
- h round to the nearest integer and to one significant figure; estimate answers to problems involving decimals
- i develop a range of strategies for mental calculation; derive unknown facts from those they know [for example, estimate  $\sqrt{85}$ ]; add and subtract mentally numbers with up to two decimal places [for example,  $13.76 - 5.21$ ,  $20.08 + 12.4$ ]; multiply and divide numbers with no more than one decimal digit [for example,  $14.3 \times 4$ ,  $56.7 \div 7$ ], using factorisation when possible

### Written methods

- j use standard column procedures for addition and subtraction of integers and decimals
- k use standard column procedures for multiplication of integers and decimals, understanding where to position the decimal point by considering what happens if they multiply equivalent fractions

- [for example,  $0.6 \times 0.7 = 0.42$  since  $6/10 \times 7/10 = 42/100 = 0.42$ ]; solve a problem involving division by a decimal by transforming it to a problem involving division by an integer
- l use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer
  - m solve simple percentage problems, including increase and decrease [for example, simple interest, discounts, pay rises]
  - n solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution [for example, given that  $m$  identical items cost  $\pounds y$ , then one item costs  $\pounds y/m$  and  $n$  items cost  $\pounds(n \times y/m)$ , the number of items that can be bought for  $\pounds z$  is  $z \times m/y$ ]

## Solving numerical problems

*Upfront* provides pupils with the opportunity to:

- a draw on their knowledge of the operations and the relationships between them, and of simple integer powers and their corresponding roots, to solve problems involving ratio and proportion, a range of measures and compound measures and metric units set in a variety of contexts
- b select appropriate operations, methods and strategies to solve number problems

## Equations, formulae and identities

*Upfront* provides pupils with the opportunity to:

### Use of symbols

- a distinguish the different roles played by letter symbols in algebra, knowing that letter symbols represent definite unknown numbers in equations [for example,  $x^3 + 1 = 65$ ], defined quantities or variables in formulae [for example,  $V = IR$ ], general, unspecified and independent numbers in identities [for example,  $3x + 2x = 5x$ , or  $3(a + b) = 3a + 3b$ , or  $(x + 1)(x - 1) = x^2 - 1$ ] and in functions they define new expressions or quantities by referring to known quantities [for example,  $y = 2 - 7x$ ]
- b understand that the transformation of algebraic expressions obeys and generalises the rules of arithmetic; simplify or transform algebraic expressions by collecting like terms [for example,  $x^2 + 3x + 5 - 4x + 2x^2 = 3x^2 - x + 5$ ], by multiplying a single term over a bracket, by taking out single term common factors [for example,  $x^2 + x = x(x + 1)$ ], and by expanding the product of two linear expressions including squaring a linear expression [for example,  $(x + 1)^2 = x^2 + 2x + 1$ ,  $(x - 3)(x + 2) = x^2 - x - 6$ ]; distinguish in meaning between the words 'equation', 'formula', 'identity' and 'expression'

### Index notation

- c use index notation for simple integer powers, and simple instances of index laws; substitute positive and negative numbers into expressions such as  $3x^2 + 4$  and  $2x^3$

### Equations

- d set up simple equations [for example, find the angle  $a$  in a triangle with angles  $a$ ,  $a + 10$ ,  $a + 20$ ]; solve simple equations [for example,  $5x = 7$ ,  $3(2x + 1) = 8$ ,  $2(1 - x) = 6(2 + x)$ ,  $4x^2 = 36$ ,  $3 = 12/x$ ], by using inverse operations or by transforming both sides in the same way

### Linear equations

- e solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution

### Formulae

- f use formulae from mathematics and other subjects [for example, formulae for the area of triangle, the area enclosed by a circle, density = mass/volume]; substitute numbers into a formula; derive a formula and change its subject [for example, convert temperatures between degrees Fahrenheit and degrees Celsius, find the perimeter of a rectangle given its area  $A$  and the length  $l$  of one side]

### Direct proportion

- g select and use equations to solve word and other problems involving direct proportion, and relate their algebraic solutions to graphical representations of the equations

### Simultaneous linear equations

- h link a graphical representation of an equation to its algebraic solution; find an approximate solution of a pair of linear simultaneous equations by graphical methods, then find the exact solution by eliminating one variable

## Sequences, functions and graphs

*Upfront* provides pupils with the opportunity to:

### Sequences

- a generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers)
- b find the first terms of a sequence given a rule arising naturally from a context [for example, the number of ways of paying in pence using only 1p and 2p coins, or from a regularly increasing spatial pattern]; find the rule (and express it in words) for the  $n$ th term of a sequence
- c generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the  $n$ th term of an arithmetic sequence

### Functions

- d express simple functions, at first in words and then in symbols; explore the properties of simple polynomial functions
- e use their knowledge of conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for  $m$  and  $c$ ) that equations of the form  $y = mx + c$  correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which  $y$  is given explicitly in terms of  $x$  [for example,  $y = 2x + 3$ ], or implicitly [for example,  $x + y = 7$ ]
- f identify and interpret graphs arising from real situations [for example, distance–time graph for an object moving with constant speed]
- g generate points and plot graphs of simple quadratic and cubic functions [for example,  $y = x^2$ ,  $y = 3x^2 + 4$ ,  $y = x^3$ ]

### Gradients

- h find the gradient of lines given by equations of the form  $y = mx + c$  (when values are given for  $m$  and  $c$ ); investigate the gradients of parallel lines and lines perpendicular to these lines [for example, knowing that  $y = 5x$  and  $y = 5x - 4$  represent parallel lines, each with gradient 5 and that the graph of any line perpendicular to these lines has gradient  $-1/5$ ]

## Ma3 Shape, space and measures

### Using and applying shape, space and measures

*Upfront* provides pupils with the opportunity to:

### Problem solving

- a select problem-solving strategies to use in geometrical work
- b select and combine known facts and problem-solving strategies to solve complex problems
- c identify what further information is needed to solve a problem; break complex problems down into a series of tasks

### Communication

- d interpret geometrical information presented in a variety of forms
- e develop an understanding of mathematical communication, making use of geometrical diagrams and related explanatory text
- f gain familiarity with the use of the precise language and exact methods to analyse geometrical configurations

### Reasoning

- k show step-by-step deduction in solving a geometrical problem

## Geometrical reasoning

*Upfront* provides pupils with the opportunity to:

### Angles

- a recall and use their knowledge of properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
- b distinguish between acute, obtuse, reflex and right angles; estimate the size of an angle in degrees

### Properties of triangles and other rectilinear shapes

- c use their knowledge of parallel lines, alternate angles and corresponding angles; recall the properties of parallelograms and that the angle sum of a triangle is 180 degrees; know that an exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices
- d use their knowledge of angle properties of equilateral, isosceles and right-angled triangles; understand congruence, recognising when two triangles are congruent; identify that the angle sum of any quadrilateral is 360 degrees
- e use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle
- f recall the essential properties of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties
- g calculate and use the sums of the interior and exterior angles of quadrilaterals
- h recall and use Pythagoras' theorem

### Properties of circles

- i recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that the tangent at any point on a circle is perpendicular to the radius at that point; recognise that the perpendicular from the centre to a chord bisects the chord

### 3-D shapes

- k use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation

## Transformations and coordinates

*Upfront* provides pupils with the opportunity to:

### Specifying transformations

- a understand that rotations are specified by a centre and an (anticlockwise) angle; use right angles, fractions of a turn or degrees to measure the angle of rotation; understand that reflections are specified by a mirror line, translations by a distance and direction

### Properties of transformations

- b recognise and visualise rotations, reflections and translations, including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform 2-D shapes by translation, rotation and reflection, recognising that these transformations preserve length and angle, so that any figure is congruent to its image under any of these transformations

### Coordinates

- e understand that one coordinate identifies a point on a number line and two coordinates identify a point in a plane; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; find the coordinates of points identified by geometrical information [for example, find the coordinates of the fourth vertex of a parallelogram with vertices at (2,1) (-7,3) and (5,6)]; find the coordinates of the midpoint of the line segment AB, given points A and B, then calculate the length AB.

## Measures and construction

*Upfront* provides pupils with the opportunity to:

### Measures

- a interpret scales on a range of measuring instruments, including those for time and mass; know that measurements using real numbers depend on the choice of unit; convert measurements from one unit to another; make sensible estimates of a range of measures in everyday settings
- c understand and use compound measures, including speed and density

### Mensuration

- f find areas of rectangles, recalling the formula, understanding the connection to counting squares and how it extends this approach; recall and use the formulae for the area of a parallelogram and a triangle; find the surface area of simple shapes using the area formulae for triangles and rectangles; calculate perimeters and areas of shapes made from triangles and rectangles
- g find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids
- h find circumferences of circles and areas enclosed by circles, recalling relevant formulae
- i convert between area measures, including  $\text{cm}^2$  and  $\text{m}^2$ , and volume measures, including  $\text{cm}^3$  and  $\text{m}^3$

### Loci

- j find loci by reasoning to produce shapes and paths [for example, equilateral triangles]

## Ma4 Handling data

### Using and applying handling data

*Upfront* provides pupils with the opportunity to:

### Problem Solving

- a carry out the first, third and fourth aspect of the handling data cycle to solve problems:
  - i specify the problem and plan: consider what inferences can be drawn from the data; decide

- what data to collect (including sample size and data format) and what statistical analysis is needed
  - iii process and represent the data: turn the raw data into usable information that gives insight into the problem.
  - iv interpret the data: answer the initial question by drawing conclusions from the data.
- b identify what further information is required to pursue a particular line of enquiry

### Communicating

- e interpret and synthesise information presented in a variety of forms
- f communicate mathematically, making use of diagrams and related explanatory text

### Reasoning

- h apply mathematical reasoning
- i explore connections in mathematics and look for cause and effect when analysing data

## Specifying the problem and planning

*Upfront* provides pupils with the opportunity to:

- b identify questions that can be addressed by statistical methods
- c understand how data relate to a problem

## Collecting data

*Upfront* provides pupils with the opportunity to:

- c use two-way tables for discrete and grouped data

## Processing and representing data

*Upfront* provides pupils with the opportunity to:

- a understand the representation of data through pie charts for categorical data and diagrams for continuous data, including line graphs for time series, scatter graphs, frequency diagrams and stem-and-leaf diagrams
- b calculate mean, range and median of small data sets with discrete then continuous data
- c understand and use the probability scale
- e list all outcomes for single events, and for two successive events, in a systematic way
- f identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
- g find the median for large data sets and calculate an estimate of the mean for large data sets with grouped data

## Interpreting results

*Upfront* provides pupils with the opportunity to:

- a relate summarised data to the initial questions
- b interpret a wide range of graphs and diagrams and draw conclusions
- c look at data to find patterns and exceptions
- d compare distributions and make inferences, using the shapes of distributions and measures of average and range
- f have a basic understanding of correlation
- h use the vocabulary of probability in interpreting results involving uncertainty and prediction



# Key Stage 4 Foundation

## Knowledge, skills and understanding

*Upfront* content draws connections between the sections on **number and algebra**, **shape, space and measures**, and **handling data**.

## Ma2 Number and algebra

### Using and applying number and algebra

*Upfront* provides pupils with the opportunity to:

#### Problem Solving

- a select and use suitable problem solving strategies and efficient techniques to solve numerical and algebraic problems
- b break down a complex calculation into simpler steps before to solve it
- c use algebra to formulate and solve a simple problem – identifying the variable, setting up an equation, solving the equation and interpreting the solution in the context of the problem
- d make mental estimates of the answers to calculations

#### Communicating

- e interpret numerical and algebraic information presented in a variety of forms
- f use notation and symbols correctly and consistently within a given problem
- h present and interpret solutions in the context of the original problem

#### Reasoning

- j explore, identify, and use pattern and symmetry in algebraic contexts
- k show step-by-step deduction in solving a problem

## Numbers and the number system

*Upfront* provides pupils with the opportunity to:

#### Integers

- a use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use positive numbers, both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple and common factor

#### Powers and roots

- b use the terms square, positive square root, negative square root, cube and cube root; use index notation for squares, cubes and powers of 10; express standard index form in conventional notation

#### Fractions

- c understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator

#### Decimals

- d use their knowledge of decimal notation and recognise that each terminating decimal is a fraction [for example,  $0.137 = 137/1000$ ]; recognise that recurring decimals are exact fractions, and that

some exact fractions are recurring decimals [for example,  $1/7 = 0.142857142857\dots$ ];  
order decimals

### Percentages

- e understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of' [for example, 10% means 10 parts per 100 and 15% of Y means  $15/100 \times Y$ ]; use percentage in real-life situations

### Ratio and Proportion

- f use ratio notation, including reduction to its simplest form and its various links to fraction notation [for example, in maps and scale drawings]

## Calculations

*Upfront* provides pupils with the opportunity to:

### Number operations and relationships between them

- a add, subtract, multiply and divide integers and then any number; multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1
- b use their knowledge of brackets and the hierarchy of operations
- c calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal
- d understand and use unit fractions as multiplicative inverses [for example, by thinking of multiplication by  $1/5$  as division by 5, or multiplication by  $6/7$  as multiplication by 6 followed by division by 7 (or vice versa)]; multiply and divide a fraction by an integer, by a unit fraction and by a general fraction
- e convert simple fractions of a whole to percentages of the whole and vice versa [for example, analysing budgets], then understand the multiplicative nature of percentages as operators [for example, a 15% increase on £150 gives a total calculated as  $\pounds(1.15 \times 150)$  while a 20% discount gives a total calculated as  $\pounds(0.8 \times 150)$ ]
- f divide a quantity in a given ratio [for example, share £15 in the ratio of 1:2]

### Mental methods

- g recall all positive integer complements to 100 [for example,  $37 + 63 = 100$ ]; recall all multiplication facts to  $10 \times 10$ , and use them to derive quickly the corresponding division facts; recall the cubes of 2, 3, 4, 5 and 10, and the fraction-to-decimal conversion of familiar simple fractions [for example,  $1/2$ ,  $1/4$ ,  $1/5$ ,  $1/10$ ,  $1/100$ ,  $1/3$ ,  $2/3$ ,  $1/8$ ]
- h round to the nearest integer and to one significant figure; estimate answers to problems involving decimals
- i develop a range of strategies for mental calculation; derive unknown facts from those they know [for example, estimate  $\sqrt{85}$ ]; add and subtract mentally numbers with up to two decimal places [for example,  $13.76 - 5.21$ ,  $20.08 + 12.4$ ]; multiply and divide numbers with no more than one decimal digit [for example,  $14.3 \times 4$ ,  $56.7 \div 7$ ] using the commutative associative and distributive laws and factorisation where possible, or place value adjustments

### Written methods

- j use standard column procedures for addition and subtraction of integers and decimals
- k use standard column procedures for multiplication of integers and decimals, understanding where to position the decimal point by considering what happens if they multiply equivalent fractions; solve a problem involving division by a decimal (up to two places of decimals) by transforming it to a problem involving division by an integer
- l use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact

answer

- m solve simple percentage problems, including increase and decrease [for example, simple interest, discounts, pay rises, income tax]
- n solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution [for example, given that  $m$  identical items cost  $\pounds y$ , then one item costs  $\pounds y/m$  and  $n$  items cost  $\pounds(n \times y/m)$ , the number of items that can be bought for  $\pounds z$  is  $z \times m/y$ ]

## Solving numerical problems

*Upfront* provides pupils with the opportunity to:

- a draw on their knowledge of the operations and the relationships between them, and of simple integer powers and their corresponding roots, to solve problems involving ratio and proportion, a range of measures and compound measures, metric units, and conversion between metric and common imperial units, set in a variety of contexts
- b select appropriate operations, methods and strategies to solve number problems

## Equations, formulae and identities

*Upfront* provides pupils with the opportunity to:

### Use of symbols

- a distinguish the different roles played by letter symbols in algebra, knowing that letter symbols represent definite unknown numbers in equations [for example,  $5x + 1 = 16$ ], defined quantities or variables in formulae [for example,  $V = IR$ ], general, unspecified and independent numbers in identities [for example,  $3x + 2x = 5x$ ,  $(x + 1)(x + 1) = x^2 + 2x + 1$  for all values of  $x$ ] and in functions they define new expressions or quantities by referring to known quantities [for example,  $y = 2x$ ]
- b understand that the transformation of algebraic expressions obeys and generalises the rules of arithmetic; manipulating algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out single term common factors [for example,  $5(2x + 3) = 10x + 15$ ;  $3x^2 + x = x(3x + 1)$ ], distinguish in meaning between the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’

### Index notation

- c use their knowledge of index notation for simple integer powers, and simple instances of index laws; substitute positive and negative numbers into expressions such as  $3x^2 + 4$  and  $2x^3$

### Linear equations

- e solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation and those with a negative solution

### Formulae

- f use their knowledge of formulae from mathematics and other subjects expressed initially in words and then using letters and symbols [for example, formulae for the area of triangle, the area enclosed by a circle, wages earned = hours worked  $\times$  rate per hour]; substitute numbers into a formula; derive a formula and change its subject [for example, convert temperatures between degrees Fahrenheit and degrees Celsius, find the perimeter of a rectangle given its area  $A$  and the length  $l$  of one side, use  $V = IR$  to generate a formula for  $R$  in terms of  $V$  and  $I$ ]

## Sequences, functions and graphs

*Upfront* provides pupils with the opportunity to:

## Sequences

- a generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the  $n$ th term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated

## Graphs of linear functions

- b use their knowledge of the conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for  $m$  and  $c$ ) that equations of the form  $y = mx + c$  correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which  $y$  is given explicitly in terms of  $x$  [for example,  $y = 2x + 3$ ], or implicitly [for example,  $x + y = 7$ ]
- c identify and use linear functions arising from real-life problems; interpret graphs arising from real situations; understand that the point of intersection of two different lines in the same two variables that simultaneously describe a real situation is the solution to the simultaneous equations represented by the lines

## Gradients

- h find the gradient of lines given by equations of the form  $y = mx + c$  (when values are given for  $m$  and  $c$ ); investigate the gradients of parallel lines

# Ma3 Shape, space and measures

## Using and applying shape, space and measures

*Upfront* provides pupils with the opportunity to:

### Problem solving

- a select problem-solving strategies to use in geometrical work
- b select and combine known facts and problem-solving strategies to solve complex problems
- c identify what further information is needed to solve a problem; break complex problems down into a series of tasks

### Communicating

- d interpret and synthesise geometrical information presented in a variety of forms
- e develop an understanding of mathematical communication explaining geometric diagrams
- f use geometrical language appropriately

### Reasoning

- i demonstrate mathematical reasoning skills
- j show step-by-step deduction in solving a geometrical problem

## Geometrical reasoning

*Upfront* provides pupils with the opportunity to:

### Angles

- a recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
- b distinguish between acute, obtuse, reflex and right angles; estimate the size of an angle in degrees

### Properties of triangles and other rectilinear shapes

- c use their knowledge of parallel lines, alternate angles and corresponding angles; recall the properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; know that

- d an exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices
- d use angle properties of equilateral, isosceles and right-angled triangles; understand congruence, recognising when two triangles are congruent; identify that the angle sum of any quadrilateral is 360 degrees
- e use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle
- f recall the essential properties of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties
- g calculate and use the sums of the interior and exterior angles of quadrilaterals
- h recall and use Pythagoras' theorem

### Properties of circles

- i recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

### 3-D shapes

- j explore the geometry of cuboids (including cubes), and shapes made from cuboids
- k use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation

## Transformations and coordinates

*Upfront* provides pupils with the opportunity to:

### Specifying transformations

- a understand that rotations are specified by a centre and an (anticlockwise) angle; rotate a shape about the origin, or any other point; measure the angle of rotation using right angles, simple fractions of a turn or degrees; understand that reflections are specified by a mirror line, at first using a line parallel to an axis, then a mirror line such as  $y = x$ ,  $y = -x$ ; understand that translations are specified by a distance and direction

### Properties of transformations

- b recognise and visualise rotations, reflections and translations, including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection, recognising that these transformations preserve length and angle, so that any figure is congruent to its image under any of these transformations

### Coordinates

- e understand that one coordinate identifies a point on a number line, two coordinates identify a point in a plane; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; find the coordinates of points identified by geometrical information [for example, find the coordinates of the fourth vertex of a parallelogram with vertices at (2, 1) (-7, 3) and (5, 6)]; find the coordinates of the midpoint of the line segment AB, given points A and B

## Measures and construction

*Upfront* provides pupils with the opportunity to:

### Measures

- a interpret scales on a range of measuring instruments, including those for time and mass; know that measurements using real numbers depend on the choice of unit; convert measurements from one unit to another; make sensible estimates of a range of measures in everyday settings
- c understand and use compound measures, including speed

### Construction

- d understand that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not

### Mensuration

- f find areas of rectangles, recalling the formula, understanding the connection to counting squares and how it extends this approach; recall and use the formulae for the area of a parallelogram and a triangle; find the surface area of simple shapes using the area formulae for triangles and rectangles; calculate perimeters and areas of shapes made from triangles and rectangles
- g find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids
- h find circumferences of circles and areas enclosed by circles, recalling relevant formulae
- i convert between area measures, including  $\text{cm}^2$  and  $\text{m}^2$ , and volume measures, including  $\text{cm}^3$  and  $\text{m}^3$

### Loci

- j find loci, by reasoning to produce shapes and paths [for example, equilateral triangles]

## Ma4 Handling data

### Using and applying handling data

*Upfront* provides pupils with the opportunity to:

- a carry out the first, third and fourth aspects of the handling data cycle to solve problems:
  - i specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed
  - iii process and represent the data: turn the raw data into usable information that gives insight into the problem
  - iv interpret: answer the initial question by drawing conclusions from the data
- b identify what further information is needed to pursue a particular line of enquiry
- c select and organise the appropriate mathematics to use for a task

### Communicating

- e interpret and synthesise information presented in a variety of forms
- f develop an understanding of mathematical communication, making use of diagrams and related explanatory text

### Reasoning

- i explore connections in mathematics and look for cause and effect when analysing data

### Specifying the problem and planning

*Upfront* provides pupils with the opportunity to:

- b identify questions that can be addressed by statistical methods
- c understand how data relate to a problem

### Processing and representing data

*Upfront* provides pupils with the opportunity to:

- a understand and recognise data represented in pie charts for categorical data, and diagrams for continuous data, including line graphs for time-series, scatter graphs, frequency diagrams and stem-and-leaf diagrams
- b calculate mean, range and median of small data sets with discrete then continuous data
- c understand and use the probability scale
- d understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
- e list all outcomes for single events, and for two successive events, in a systematic way
- f identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
- g find the median for large data sets and calculate an estimate of the mean for large data sets with grouped data

### Interpreting results

*Upfront* provides pupils with the opportunity to:

- a relate summarised data to the initial questions
- b interpret a wide range of graphs and diagrams and draw conclusions
- c look at data to find patterns and exceptions
- d compare distributions and make inferences, using the shapes of distributions and measures of average and range
- f have a basic understanding of correlation as a measure of the strength of the association between two variables
- g use the vocabulary of probability to interpret results involving uncertainty and prediction
- h compare experimental data and theoretical probabilities
- j recognise implications of findings in the context of the problem
- k interpret social statistics, time series [for example, population growth] and survey data

# Key Stage 4 Higher

## Knowledge, skills and understanding

*Upfront* content draws connections between the sections on **number and algebra**, **shape, space and measures**, and **handling data**.

## Ma2 Number and algebra

### Using and applying number and algebra

*Upfront* provides pupils with the opportunity to:

#### Problem Solving

- a select and use appropriate and efficient techniques to solve numerical or algebraic problems of increasing complexity, involving numerical and algebraic manipulation
- b identify what further information may be required in order to pursue a particular line of enquiry
- c breaking down a complex calculation into simpler steps before attempting a solution
- d make mental estimates of the answers to calculations

#### Communicating

- h use notation and symbols correctly and consistently within a given problem

#### Reasoning

- j identify, and use pattern and symmetry in algebraic contexts
- l show step-by-step deduction in solving a problem

## Numbers and the number system

*Upfront* provides pupils with the opportunity to:

#### Integers

- a use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor (decomposition)

#### Powers and roots

- b use the terms square, positive square root, negative square root, cube and cube root; use index notation [ $8^2$ ,  $8^{2/3}$ ] and index laws for multiplication and division of integer powers; use standard index form, expressed in conventional notation

#### Fractions

- c understand equivalent fractions, simplify a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator

#### Decimals

- d recognise that each terminating decimal is a fraction [for example,  $0.137 = 137/1000$ ]; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals [for example,  $1/7 = 0.142857142857\dots$ ]; order decimals

## Percentages

- e understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of' [for example, 10% means 10 parts per 100 and 15% of Y means  $15/100 \times Y$ ]

## Ratio and Proportion

- f use ratio notation, including reduction to its simplest form and its various links to fraction notation

## Calculations

*Upfront* provides pupils with the opportunity to:

### Number operations and relationships between them

- a multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer, fractional and negative powers; use inverse operations, understanding that the inverse operation of raising a positive number to power  $n$  is raising the result of this operation to power  $1/n$
- b use brackets and the hierarchy of operations
- c calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal; distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals); convert a recurring decimal to a fraction [for example,  $0.142857142857 = 1/7$ ]
- d understand and use unit fractions as multiplicative inverses [for example, by thinking of multiplication by  $1/5$  as division by 5, or multiplication by  $6/7$  as multiplication by 6 followed by division by 7 (or vice versa)]; multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction
- e convert simple fractions of a whole to percentages of the whole and vice versa [for example, analysing budgets], then understand the multiplicative nature of percentages as operators [for example, a 15% increase on £Y gives a total calculated as  $\pounds(1.15 \times Y)$ ]; calculate an original amount when given the transformed amount after a percentage change; reverse percentage problems [for example, a ticket to the football cost £20 last season, this season there has been a 12% increase in the cost of the ticket, calculate the new cost  $\pounds 20/1.12$ ]
- f divide a quantity in a given ratio

### Mental methods

- g recall integer squares from  $2 \times 2$  to  $15 \times 15$  and the corresponding square roots, the cubes of 2, 3, 4, 5 and 10 and simple cube roots
- h round to a given number of significant figures; develop a range of strategies for mental calculation; derive unknown facts from those they know; convert between ordinary and standard index form representations [for example,  $0.1234 = 1.234 \times 10^{-1}$ ], converting to standard index form to make sensible estimates for calculations involving multiplication and/or division

### Written methods

- i use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation
- j solve percentage problems, including percentage increase and decrease [for example, simple interest]; and reverse percentages

- k represent repeated proportional change using a multiplier raised to a power
- l calculate an unknown quantity from quantities that vary in direct or inverse proportion
- m calculate with standard index form [for example,  $2.4 \times 10^7 \times 5 \times 10^3 = 12 \times 10^{10} = 1.2 \times 10^{11}$ ,  $(2.4 \times 10^7) \div (5 \times 10^3) = 4.8 \times 10^3$ ]
- n use surds and  $\pi$  in exact calculations, without a calculator; rationalise a denominator such as  $1/\sqrt{3} = \sqrt{3}/3$

## Solving numerical problems

*Upfront* provides pupils with the opportunity to:

- a draw on their knowledge of the operations and inverse operations (including powers and roots), and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, repeated proportional change, fractions, percentages and reverse percentages, inverse proportion, surds, measures and conversion between measures, and compound measures defined within a particular situation

## Equations, formulae and identities

*Upfront* provides pupils with the opportunity to:

### Use of symbols

- a distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations [for example,  $x^2 + 1 = 82$ ], defined quantities or variables in formula [for example,  $V = IR$ ], general, unspecified and independent numbers in identities [for example,  $(x + 1)^2 = x^2 + 2x + 1$ , for all  $x$ ] and in functions they define new expressions or quantities by referring to known quantities [for example,  $y = 2 - 7x$ ;  $f(x) = x^3$ ;  $f(x) = 1/x$  for  $x \neq 0$ ]
- b understand that the transformation of algebraic entities obeys and generalises the well-defined rules of arithmetic; expand the product of two linear expressions; manipulating algebraic expressions by collecting like terms, multiplying a single term over a bracket, taking out common factors [for example,  $9x^2 - 3x = 3x(3x - 1)$ ], factorising quadratic expressions including the difference of two squares [for example,  $x^2 - 9 = (x + 3)(x - 3)$ ] and cancelling common factors in rational expressions [for example,  $2(x + 1)^2/(x + 1) = 2(x + 1)$ ]

### Index notation

- d use index notation for simple integer powers, and simple instances of index laws [for example,  $x^3 \times x^2 = x^5$ ;  $x^2/x^3 = x^{-1}$ ;  $(x^2)^3$ ] substitute positive and negative numbers into expressions such as  $3 \times 2 + 4$  and  $2 \times 3$

### Equations

- e set up simple equations [for example, find the angle  $a$  in a triangle with angles  $a$ ,  $a + 10$ ,  $a + 20$ ]; solve simple equations [for example,  $5x = 7$ ;  $11 - 4x = 2$ ;  $3(2x + 1) = 8$ ;  $2(1 - x) = 6(2 + x)$ ;  $4x^2 = 49$ ;  $3 = 12/x$ ] by using inverse operations or by transforming both sides in the same way

### Linear equations

- f solve linear equations in one unknown, with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation and those with a negative solution

### Formulae

- g use formulae from mathematics and other subjects [for example, for area of a triangle or a parallelogram, area inclosed by a circle, volume of a prism, volume of a cone]; substitute numbers into a formula; change the subject of the formula, including cases where the subject occurs twice, or where a power of a subject appears [for example, find  $r$  given  $A = \pi r^2$ , find  $x$  given  $y = mx + c$ ]; generate a formula [for example, find the perimeter of a rectangle given its area  $A$  and the length  $l$  of one side]

### Simultaneous equations

- i find the exact solution of two simultaneous equations in two unknowns by eliminating a variable, and interpret the equations as lines and their common solution as the point of intersection

## Sequences, functions and graphs

*Upfront* provides pupils with the opportunity to:

### Sequences

- a generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); generate terms of a sequence using term-to-term or position-to-term definitions of the sequence; use linear expressions to describe the  $n$ th term of an arithmetic sequence

### Graphs of linear functions

- b use conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for  $m$  and  $c$ ) that equations of the form  $y = mx + c$  correspond to straight-line graphs in the coordinate plane; identify graphs of functions in which  $y$  is given explicitly in terms of  $x$  [for example,  $y = 2x + 3$ ], or implicitly [for example,  $x + y = 7$ ]
- c find the gradient of lines given by equations of the form  $y = mx + c$  (when values are given for  $m$  and  $c$ ); understand that the form  $y = mx + c$  represents a straight line and that  $m$  is the gradient of the line, and  $c$  is the value of the  $y$  intercept

### Interpreting graphical information

- d interpret and recognise graphs arising from real situations [for example, distance-time graph for a particle moving with constant velocity, the depth of water in a container as it empties]

### Other functions

- f identify graphs of: simple cubic functions [for example,  $y = x^3$ ], the reciprocal function  $y = 1/x$  with  $x \neq 0$ , the exponential function  $y = kx$  for integer values of  $x$  and simple positive values of  $k$  [for example,  $y = 2x$ ], the circular function  $x^2 + y^2 = r^2$

## Ma3 Shape, space and measures

### Using and applying shape, space and measures

*Upfront* provides pupils with the opportunity to:

### Problem solving

- a select problem-solving strategies, to use in geometrical work
- b select and combine known facts and problem-solving strategies to solve complex geometrical problems

## Communication

- e select precise and formal language and exact methods for analysing geometrical configurations

## Reasoning

- j understand the necessary and sufficient conditions under which solutions to geometrical problems remain valid.

## Geometrical reasoning

*Upfront* provides pupils with the opportunity to:

### Properties of triangles and other rectilinear shapes

- a use parallel lines, alternate angles and corresponding angles; recall the properties of parallelograms and that the angle sum of a triangle is 180 degrees; know that an exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices
- b use angle properties of equilateral, isosceles and right-angled triangles; identify the angle sum of any quadrilateral as 360 degrees
- c recall the definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties
- d calculate the sums of interior and exterior angles of quadrilaterals
- e understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments
- f recall and use Pythagoras' theorem
- g understand similarity of triangles

### Properties of circles

- h recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; know that the tangent at any point on a circle is perpendicular to the radius at that point; recognise the fact that tangents from an external point are equal in length; know that the perpendicular from the centre to a chord bisects the chord; recall that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, that the angles in the same segment are equal, and that the opposite angles of a cyclic quadrilateral sum to 180 degrees
- i use 2-D representations of 3-D shapes and analyse 3-D solids through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms, pyramids, cylinders, cones and spheres.

## Transformations and coordinates

*Upfront* provides pupils with the opportunity to:

### Specifying transformations

- a understand that rotations are specified by a centre and an (anticlockwise) angle; use any point as the centre of rotation; measure the angle of rotation, using right angles, fractions of a turn or degrees; understand that reflections are specified by a (mirror) line; understand that translations are specified by giving a distance and direction

### Properties of transformations

- b recognise and visualise rotations, reflections and translations, including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations

## Coordinates

- e understand that one coordinate identifies a point on a number line and that two coordinates identify a point in a plane; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates, find the coordinates of points identified by geometrical information; find the coordinates of the midpoint of the line segment AB, given the points A and B, then calculate the length AB

## Measures and construction

*Upfront* provides pupils with the opportunity to:

### Measures

- a know that measurements using real numbers depends on the choice of unit; convert measurements from one unit to another; understand and use compound measures including speed

### Mensuration

- d find the surface area of simple shapes by using the formulae for the areas of triangles and rectangles; find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of cuboids, recalling the formula and approach; calculate volumes of right prisms and of shapes made from cubes and cuboids; convert between volumes of right prisms and of shapes made from cubes and cuboids; convert between volume measures including  $\text{cm}^3$  and  $\text{m}^3$ ; find circumferences of circles and areas enclosed by circles, recalling relevant formulae

### Loci

- j find loci, by reasoning to produce shapes and paths [for example, equilateral triangles]

## Ma4 Handling data

### Using and applying handling data

*Upfront* provides pupils with the opportunity to:

#### Problem solving

- a carry out the first, third and fourth aspects of the handling data cycle to solve problems:
  - i specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed)
  - iii process and represent the data: turn the raw data into usable information that gives insight into the problem
  - iv interpret and answer the initial question by drawing conclusions from the data

#### Communicating

- c communicate mathematically

### Processing and representing data

*Upfront* provides pupils with the opportunity to:

- a identify the correct pie chart for categorical data; identify the diagrams from continuous data, including the line graph, scatter graphs, frequency diagrams, stem-and-leaf diagrams, cumulative frequency tables and histograms for grouped continuous data
- b understand measures of probability
- c list all outcomes for single events in a systematic way

- d identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
- e find the median for large data sets and calculate the mean for large data sets with grouped data
- g know when to add or multiply two probabilities: if A and B are mutually exclusive, then the probability of A and B occurring is  $P(A) + P(B)$ , whereas if A and B are independent events, the probability of A and B occurring is  $P(A) \times P(B)$

### Interpreting results

*Upfront* provides pupils with the opportunity to:

- a relate summarised data to the initial questions
- b interpret a wide range of graphs and diagrams and draw conclusions
- d compare distributions and make inferences, using the shapes of distributions and measures of average and spread
- g use the vocabulary of probability to interpret results involving uncertainty and prediction.